AGI vrs Average Area

Math 300 Spring 2003 [1]

This is a description of some of the work done in class on April 7. It is expected from 1/3 to 1/2 of Essay 3 will be devoted to issues like those raised here. But, some of you may address from different standpoints and certainly with different examples.

Suppose there are two samples of very regular cross sections of crystals. In Sample A each cross section is a 2cm by 2cm square. In Sample B each cross section is a 1cm by 4cm rectangle. (A sheet showing this is available but not online.) We earlier asked the following questions. We give the answers below.

1. Compute the average 'crystal area' for each samples A and B.

2. Do sample A and sample B have the same AGI. Hint: give estimates of (upper and lower bounds on) the average grain indicator for each of samples A and B for various kinds of lines.

   (a) What is the AGI for horizontal lines of length 3.2 cm in each of samples A and B?

   (b) What is the AGI for vertical lines of length 3.2 cm in each of samples A and B?

   (c) What is the AGI for 45^o lines of length 3.2 cm in each of samples A and B?

For simplicity in computing the probability, I switch to lines 3 cm. long and ignore what happens if the either end of the line is exactly on the boundary between two crystal. (The probability of such an event is 0).

1. Both Sample A and sample B have average area 4 sq cm.

2. Sample A Squares: The AGI for horizontal lines of length 3 cm in sample A is $.5(1) + .5(2) = 1.5$. This holds because if the left end of a 3 cm horizonal line is in the left half of a square the line crosses only the right side of the square. If the left end of 3 cm horizonal line is in the right half of a square the line crosses both the right side of the square it is in and the right side of the next square.

3. The AGI for vertical lines in a square is 1.5 for exactly the same reason (reasoning vertically instead of horizontally).

[1] Adapted from materials developed by the Small Group Mathematical Modeling (SGMM) Project, Purdue University.

4. Sample B rectangle: The AGI for horizontal lines of length 3 cm in sample B is $.25(0) + .75(1) = .75$. This holds because if the left end of a 3 cm horizonal line is in the left quarter of a rectangle the line doesn't cross any boundary. If the left end of a 3 cm horizonal line is in the right three-quarters of a rectangle the line crosses the right side of the rectangle.

5. Sample B rectangle: The AGI for vertical lines of length 3 cm in sample B is 3 because every vertical line of length 3 cm intersects 3 horizontal boundaries. (Again we omit the unlikely case that it starts and stops on a boundary.)

6. Now suppose half the lines are vertical and half horizontal. Then for sample A we get an AGI of $.5(1.5)+.5(1.5) = 1.5$. But for sample B we get an AGI of $.5(.75)+.5(3) = 1.875$.

7. I haven't written out the 45^o case but it is similar.

We conclude that samples with the same average area can have different AGI. (You may want to challenge that conclusion because I only considered two directions of lines. You may want to argue for the conclusion and you may want to try some other directions to support such an argument.)

You may want to try this with different lengths or different directions of the lines. Or you might consider some triangular cross-sections as we discussed in class with the geoboards.

You don't have to deal with these specific examples. But, in the second half of your essay you need to address the question. Do differences in shape make the 'average diameter' (AGI) a more precise indicator than the average area?